STUDY OF FLOW DYNAMICS BY OPTICAL-CORRELATION METHOD

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We examine the possibility of studying turbulent pulsations of the flow velocity by correlation and spectral analysis of optical signals from two or more points of a stream.

1. **Convective velocity measurements.** Correlation analysis has been widely used for measuring the flow velocity by means of various probes: hot-wire anemometers [1–3], inductive sensors [4], pressure sensors [3], and so on. The convective flow velocity is found by correlation analysis of the signals obtained by these probes.

By convective velocity we usually mean the rate of transport of the components of some field: pressure, temperature, conductivity, or the velocity field itself. We can also speak of the convective velocity of the optical fluctuation field.

If we record the fluctuations of some physical quantity at two points located along the stream direction, the time shift of the peak of the corresponding cross-correlation function is customarily identified with the characteristic transport time of this quantity from one measurement point to the other. If the pulsations are stationary the shift will correspond to the average nonhomogeneity travel time, i. e., their local-average transport velocity. If the fluctuations are unsteady, fluctuations of larger amplitude yield a larger contribution to the shift of the cross-correlation curve and thereby determine the magnitude of the measured convective velocity.

This technique for determining the velocity was used initially for measuring the convective velocity of the velocity field itself, starting from the Taylor hypothesis on "frozen" turbulence. In accordance with this hypothesis, if the turbulence intensity is not high the velocity pulsations at some point in the flow will be the result of transport through the given point with the convective velocity of the spatial signature of the stream (here signature means the velocity field itself), i. e.,

$$v(\mathbf{x}, t) = v(\mathbf{x} - \mathbf{V}t, 0) \tag{1.1}$$

where $\mathbf{x}$ is the coordinate of the point and $\mathbf{V}$ is the convective velocity. In this case the cross-correlation function of the velocity pulsations at two points located at a distance L from one another along the stream has the form

$$R(\tau) = \langle v(\mathbf{x}, t)\, v(\mathbf{x} + \mathrm{L}, t + \tau)\rangle = \langle v(\mathbf{x}, t)\, v(\mathbf{x}, t - L V_L^{-1} + \tau)\rangle \tag{1.2}$$

It is obvious that for $\tau = \mathrm{L}/\mathrm{V_L}$ the velocity pulsations at the two points will be referred to the same phase and the cross-correlation function will reach its maximum value. Hence follows directly the technique for determining the convective velocity from the time shift of the peak of the velocity-pulsation cross-correlation function at two points.

However, Lin showed on the basis of the Navier-Stokes equations that the Taylor hypothesis is valid only if the turbulence intensity, viscous forces, and tangential stresses are small. Otherwise it is necessary to account for the influence of the average and fluctuating tangential stresses, which alter the turbulence structure continuously. Under these conditions we can no longer speak of some constant convective velocity. It has been shown by several authors [1–4] that we can speak only of the convective velocity $V(\mathbf{k})$ of some definite wavenumber. Thus, if the velocity field is considered as the finite sum of uncorrelated plane waves of different orientation and length with random amplitudes and phases, then the convective velocity V_c of a specific wave number can be defined as its most probable transport velocity, i. e., for $V = V_c$ the spectral density function W(k, V) reaches its peak,

$$\left\{\frac{\partial W(k, V)}{\partial V}\right\}_{V=V_c} = 0 \tag{1.3}$$

It is obvious that in hot-wire measurements this velocity will to some degree correspond to the time shift of the peak of the cross-correlation function of filtered signals of definite frequency (more precisely, some finite narrow frequency band).

In this case the over-all convective velocity V_* can be determined as the value for which

$$\left\{\frac{\partial W(V)}{\partial V}\right\}_{V=V_*}=0, \qquad W(V)=\int_{-\infty}^{\infty} W(k, V)\, dk$$

This velocity will not necessarily correspond to the shift of the peak of the over-all (unfiltered) cross-correlation curve, since as we have noted previously this shift may be determined not by the transport velocity of the most probable wave numbers but primarily by the transport velocity of the wave numbers corresponding to the pulsations with maximum amplitude.

The correlation method was later used to measure the transport velocity of other physical fields, e.g., the pressure fields [3].

In [4] a similar method was used to measure the convective velocity of the quantity σV in a plasma jet (σ is the plasma electrical conductivity; V is velocity). The σV fluctuations were detected with the aid of currents induced by the fluctuations in measuring coils as the plasma traveled in an external magnetic field.

The trend toward remote measurement methods, which do not introduce disturbances into the stream, and also the trend toward increased spatial resolution, has led to the use of the correlation method for optical measurements. In this case the signal sources can be fluctuations of the refraction coefficient, brightness, density, impurity concentration, and so on. The transport of the optical nonhomogeneities is identified with the transport of the gas stream itself.

For example, in [5, 6] recordings were made of natural fluctuations of the luminosity of a plasma jet at two axial points, and the shift of the cross-correlation function peak was used to calculate the propagation velocity of these fluctuations.

In [7] the convective velocity of the optical fluctuation field was determined from the shift of the peak of the cross-correlation function of signals from two transmitted beams which were absorbed by the medium.

In order for absorption to take place, liquid nitrogen or water was sprayed into the stream to form a finely dispersed fog. The beams were directed perpendicular to one another and to the flow axis, and the distance between the beams in the flow direction was varied. The cross-correlation function of the signals from two photodetectors was found for different distances between the beams.

Thus, the correlation technique has been used in quite varied measurements as a technique for finding the convective velocity of the quantities being measured. The primary characteristic is the time shift of the peak of the cross-correlation function. The resulting value is the mathematical expectation of the velocities and does not yield any representation of the scatter of the velocities around this value.

At the same time the cross-correlation function spectrum contains more complete information on the flow velocity than can be obtained from the shift of the function peak alone.

The following section is devoted to examination of this question.

2. **Effect of velocity pulsations on the frequency spectrum of optical fluctuations.** We consider the random uniform isotropic field $N(\mathbf{x}, t)$ of the optical nonhomogeneities of the medium in question, whose transport we identify with the motion of the fluid itself under the assumption that it is incompressible.

As is known [8], such a field can be represented in the form of the superposition of waves of different orientation with random amplitude and phase:

$$N(\mathbf{x})=\int e^{i\mathbf{k}\mathbf{x}}\, dZ(\mathbf{k}) \tag{2.1}$$

Here $\mathbf{k}$ is the wave vector and Z is the complex amplitude and the integral extends over the entire wave vector space. The temporal variations of the field can be described on the basis of the corresponding transport model.

In the case of the frozen turbulence model, now applied to the optical nonhomogeneity field, all the time variations of N(**x**, t) at a point of the stream are caused by transport of the spatial field distribution with constant velocity. However, such a model does not allow us to find the velocity pulsations in the stream.

It is obvious that the more general model will be that in which the transport of the frozen field takes place with a time-variable velocity. In this case we obtain in place of (1.1)

$$N(\mathbf{x}, t) = N\left(\mathbf{x} - \int_0^t \mathbf{v}(\tau)\, d\tau,\, 0\right) \tag{2.2}$$

With account for the given transport conditions, (2.1) takes the form

$$N(\mathbf{x}, t) = \int \exp\left[i\mathbf{k}\int_0^t \mathbf{v}(\tau)\, d\tau\right] dZ(\mathbf{k}) \tag{2.3}$$

If in this case $\mathbf{v}(\tau) = \langle \mathbf{v} \rangle + \mathbf{v}'(\tau)$, where $\langle \mathbf{v} \rangle$ is the average velocity and $\mathbf{v}'(\tau)$ is the pulsative component, then

$$N(\mathbf{x}, t) = \int \exp\left\{i\mathbf{k}\left[\langle \mathbf{v} \rangle t + \int_0^t \mathbf{v}'(\tau)\, d\tau\right]\right\} dZ(\mathbf{k}) \tag{2.4}$$

It follows from (2.4) that N(x, t) is the result of frequency modulation by the random process v'(t) of the random function $\varepsilon(\mathbf{x}, t)$, equal to

$$\varepsilon(\mathbf{x}, t) = \int e^{i\mathbf{k}\langle \mathbf{v} \rangle t}\, dZ(\mathbf{k}) \tag{2.5}$$

This function corresponds to the signal without velocity pulsations (model of frozen field transported with constant velocity). The existence of velocity pulsations leads to the appearance of frequency modulation. In this case the frequency deviation of each elementary harmonic of the process $\varepsilon(x, t)$ is proportional to the mean-square velocity pulsation $\sqrt{\langle v_x^2 \rangle}$ in the x-direction, and the effective modulation frequency is determined by the velocity pulsation correlation interval, i. e., by the Euler integral time scale [9].

In fact, the Kampe de Feriet relation

$$\langle \xi^2 \rangle = \left\langle \left[\langle v_x \rangle t + \int_0^t v_x'(\tau)\, d\tau\right]^2 \right\rangle = 2\langle v_x^2 \rangle \int_0^t (t - \tau) R_{v_x}(\tau)\, d\tau \tag{2.6}$$

is valid for the coordinate variance in (2.4). In (2.6) $R_{v_x}(\tau)$ is the correlation coefficient between the velocity pulsations at different times and $\langle v_x^2 \rangle$ is the velocity variance.

Consequently, the amplitude of the modulating function will be the magnitude of the mean square velocity pulsation, and the frequency deviation of each i-th elementary harmonic of the process $\varepsilon(x, t)$ can be found from the formula

$$\Delta\omega_i = \frac{2\pi}{\lambda_i}\sqrt{\langle v_x^2 \rangle} = \frac{\omega_i}{\langle v_x \rangle}\sqrt{\langle v_x^2 \rangle} \tag{2.7}$$

However, the effective modulation period in the model adopted will obviously be determined by the ratio of the space scale Λ, for which particle motion takes place essentially only in one direction, to the magnitude of the mean-square velocity pulsation

$$T_m = \Lambda / \langle v_x^2 \rangle^{1/2} = \Lambda \langle v_x^2 \rangle^{1/2} / \langle v_x^2 \rangle \tag{2.8}$$

But the numerator on the right-hand side of (2.8) is simply the eddy diffusion coefficient D, which can be found from (2.6) by passing to the limit as $t \to \infty$ [10]. Then

$$D = \langle \xi^2(t) \rangle / 2t = \langle v_x^2 \rangle \mathcal{T}_E \tag{2.9}$$

where $\mathcal{T}_E$ is the Euler integral time scale.

Substituting (2.9) in place of the numerator in the right-hand side of (2.8), we obtain

$$T_m = \mathcal{T}_E \quad \text{or} \quad \Omega_m = 1 / \mathcal{T}_E \tag{2.10}$$

Thus the effective modulation frequency is determined directly by the Euler integral time scale.

This has definite physical meaning, since the magnitude of the integral time scale can serve as a measure of the longest time interval in the course of which transport of the field space distribution takes place on the average in the given direction, i.e., it will be the maximum modulation period.

It follows from (2.10) that the larger the integral time scale the lower the effective modulation frequency. In the limiting case, when $\mathscr{T}_E = \infty$, $\Omega_m = 0$, i.e., there is no modulation (transport of the frozen field with constant velocity). On the other hand, when $\mathscr{T}_E \to 0$, $\Omega_m \to \infty$, i.e., when the nature of the velocity pulsations approaches white noise, the modulation frequency becomes infinitely large.

Thus, on the basis of the model of the frozen field transported with variable velocity we can conclude that the recording of the optical fluctuations includes information on the turbulence in the form of the modulating function.

3. Methods for separating the turbulence characteristics from the optical fluctuation records. In accordance with the adopted transport model, the variation of the frequency of each elementary harmonic of the modulated process $\varepsilon(x, t)$ is defined by the same modulating function $v_x'(t)$, i.e.,

$$\omega_j(t) = \omega_j + \frac{2\pi}{\lambda_j} v_x'(t) \tag{3.1}$$

Then the elementary function has the form

$$y_j = V(\omega_j)\cos\left[\omega_j t + \int_0^t \frac{2\pi}{\lambda_j} v_x'(t)\,dt\right] + iV(\omega_j)\sin\left[\omega_j t + \int_0^t \frac{2\pi}{\lambda_j} v_x'(t)\,dt\right] \tag{3.2}$$

Let

$$v_x'(t) = \sum_{\nu=-N}^{N} V_\nu e^{i\Omega_\nu t}, \quad V_\nu = \int_{\Omega_\nu-\alpha}^{\Omega_\nu+\alpha} V(\Omega)\,d\Omega \quad (\nu = 0, \pm 1, \ldots \pm N)$$

Here $V(\Omega)$ is white noise. Then

$$\begin{aligned} y_j = V(\omega_j)\left[\cos\omega_j t - \sum_{\nu=-N}^{N} \beta_\nu e^{i\Omega_\nu t}\sin\omega_j t\right] + iV(\omega_j) \\ \times\left[\sin\omega_j t + \sum_{\nu=-N}^{N} \beta_\nu e^{i\Omega_\nu t}\cos\omega_j t\right], \quad \beta_\nu = \frac{2\pi V_\nu}{\lambda_j \Omega_\nu} \end{aligned} \tag{3.3}$$

under the assumption of smallness of β, equal to the sum of the β_ν.

Performing the summation over all the elementary harmonics, we find the expression for the FM signal for the case of modulation of one random process by another random process with small β:

$$\begin{aligned} X(t) &= \int_{-\infty}^{\infty} V(\omega_j) e^{i\omega_j t}\,d\omega_j - \frac{1}{i}\int_{-\infty}^{\infty} V(\omega_j)\sum_\Omega \beta e^{i\Omega t} e^{i\omega_j t}\,d\omega_j \\ &= \varepsilon(t)\left[1 + \sum_\Omega \frac{2\pi}{\lambda}\frac{V(\Omega)}{\Omega} e^{i\Omega t}\right] = \varepsilon(t)\left[1 + v_x^*(t)\right] \\ & v_x^*(t) = 2\pi v_x(t) / \lambda\Omega \end{aligned} \tag{3.4}$$

Thus, frequency modulation of the random process $\varepsilon(t)$ by the random process $v_x(t)$ in the case of small β reduces to the product of $\varepsilon(t)$ by $v_x^*(t)$. The latter is some dimensionless velocity whose statistical characteristics, however, will be completely identical to the statistical characteristics of $v_x(t)$. Thus, we can say that the modulation in this case reduces to the product of the random processes themselves, i.e., the modulation is of the amplitude modulation type.

Let us consider what the condition of smallness of β means. Since $2\pi / \lambda = \omega_j / \langle v_x \rangle$, we obtain

$$\sum \frac{\omega_j}{\Omega}\frac{V(\Omega)}{\langle v_x\rangle} \ll 1 \tag{3.5}$$

The first fraction under the summation sign is the ratio of the modulated harmonic frequency to the modulating frequency, and the second is the ratio of the amplitude of the corresponding velocity pulsation harmonic to the average stream velocity. The first fraction characterizes the relationship between the average dimensions of the optical nonhomogeneities and the eddies, the second represents the intensity of the turbulent velocity pulsations. Thus, the assumption indicated above imposes a corresponding limitation on the ratio of the dimensions of the optical nonhomogeneities to the dimension of the eddies as a function of the turbulence intensity. For low turbulence intensity the ratio of the average dimensions of the nonhomogeneities to the dimensions of the eddies can be large, and conversely, in the presence of high turbulence intensity the dimensions of the optical nonhomogeneities must be less than the eddy dimensions.

Expression (3.4) makes it possible to find the optical fluctuation spectrum at a fixed stream point in terms of the spectra of the modulated and modulating processes.

In fact, if the processes $\varepsilon(t)$ and $v_x(t)$ are statistically independent the correlation function X(t) of the frequency-modulated process will be

$$K_{xx}(\tau) = K_{\varepsilon\varepsilon}(\tau) + K_{v^*v^*}(\tau) K_{\varepsilon\varepsilon}(\tau) \tag{3.6}$$

Hence

$$S_x(\omega) = S_\varepsilon(\omega) + \frac{1}{\pi} \int_{-\infty}^{\infty} S_{v^*}(\Omega) S_\varepsilon(\omega - \Omega) \, d\Omega \tag{3.7}$$

Since $S_{v^*}(\Omega)$ and $S_v(\Omega)$, after being normalized, are identical, we can using (3.7) separate the velocity pulsation spectrum if $S_\varepsilon(\omega)$ is known, and the latter, as will be shown later, can be found by using the structure function.

Thus, in the case of ideal realization of the adopted frozen-field model the single-point correlation and the corresponding spectrum can be used to identify the turbulence characteristics. However, velocity pulsations having a different direction with relation to the average velocity vector modulate the basic process differently. We can identify the influence of pulsations having a given direction (longitudinal pulsations, for example) by processing the signals from two points. In this case the cross-correlation function identifies the frequency-modulated signal in the given direction, and therefore the cross-correlation function spectrum will carry information on both the magnitude of the mean-square velocity pulsation in the given direction and the one-dimensional spectra of the modulated and modulating processes [6]. Moreover, the structure function of the signals from two points of the stream identifies the modulated signal in the given direction and its spectrum will correspond to the $S_\varepsilon(\omega)$ spectrum.

In fact, let us assume that at the first point of the stream there is recorded the signal $X_1(t) = [1 + v_x^*(t)]\varepsilon(t)$, and at the second point the same signal shifted in time by the amount η:

$$X_2(t) = [1 + v_x^*(t + \eta)]\varepsilon(t + \eta)$$

The time shift η is a result of the fact that a particular nonhomogeneity, after passing the first point with the velocity corresponding to the field transport velocity at the time t, passes the second point with the velocity corresponding to the transport velocity at the time $(t + \eta)$.

Then

$$\begin{aligned} D_{x_1x_2}(\eta) &= \langle [X_1(t) - X_2(t + \eta)]^2 \rangle \\ &= 2\,[K_{\varepsilon\varepsilon}(0) + K_{\varepsilon\varepsilon}(0) K_{v^*v^*}(0)] - 2K_{\varepsilon\varepsilon}(\eta)\,[1 + K_{v^*v^*}(\eta)] \end{aligned} \tag{3.8}$$

We see from (3.8) that velocity pulsations which are slow in comparison with η have no effect on the structure function. In this case, if the spectral density of the velocity pulsations with frequency higher than $1/\eta$ is negligibly small the structure-function spectrum will correspond to the spectrum of the stream "signature" pulsations, i. e., it will correspond to the spectrum of the frozen field transported with constant velocity.

Moreover, it follows from (3.8) that in the absence of velocity pulsations the cross-correlation function coincides with the structure function to within a constant factor. Thus, comparison of the spectra of the structure and cross-correlation functions makes it possible to establish the presence or absence of the modulation effect owing to velocity pulsations.

4. Measured parameters. The following assumptions were made in the method in question for the measurement of turbulence:

1) the optical nonhomogeneity field is frozen within the limits of the measurement baseline;

2) the recorded optical fluctuations are stationary functions in the course of the realization time;

3) the velocity pulsations are slowly varying time functions in comparison with the modulated process (signature of the frozen field, transported with constant velocity);

4) the optical nonhomogeneity field and the velocity field are statistically independent.

As noted previously, under these assumptions the average stream velocity is determined by the time corresponding to the shift of the peak of the cross-correlation function of the optical signals from the two stream points.

The frequency deviation in the cross-correlation-function spectrum is proportional to the mean-square velocity pulsation. If we assume that the cross spectral density function $S_{xy}(\omega)$ repeats the frequency probability density distribution law for the frequency-modulated signal in a given direction, then the magnitude of the effective deviation $\Delta\omega_{eff}$ can be calculated using the formula

$$\Delta\omega_{eff} = \left[\int_{-\infty}^{\infty} (\omega - \langle\omega\rangle)^2 S_{xy}(\omega)\, d\omega\right] \left[\int_{-\infty}^{\infty} S_{xy}(\omega)\, d\omega\right]^{-1} \tag{4.1}$$

The latter is in turn related with the magnitude of the mean-square velocity pulsation by (2.7).

Since the most probable (carrier) frequency of the modulated process is proportional to the average stream velocity, the ratio of the frequency deviation to the carrier frequency yields directly the relative intensity of the turbulence, i. e.,

$$\frac{\Delta\omega}{\omega_0} = \frac{\langle v_x^2(t)\rangle^{1/2}}{\langle v_x\rangle} \tag{4.2}$$

Relation (3.7), considered as an expression for the spectrum of the frequency-modulated signal in a given direction (to which corresponds the cross spectral density of the optical signals from the two points), makes it possible to identify the spectrum of the velocity pulsations in the given direction. In this case, as indicated above, it is advisable to compare $S_{xy}(\omega)$ with the structure-function spectrum $S_\varepsilon(\omega)$ in order to refine the boundaries of the modulated process.

Along with the local flow characteristics, identifiable for a given flow direction by analysis of the signals from two points of the stream, if there are a large number of measurement points it is obviously possible to obtain such parameters as turbulent stresses, flow-field time-variation characteristics, average eddy lifetime, and so on.

5. Examples of experimental results. Figure 1 shows oscillograms representing the intensity of the light scattered by the natural microscopic suspended matter present in a water stream. The stream is illuminated by a light beam from a helium-neon laser directed along the stream. The scattered radiation is recorded from two flow regions with linear dimension of order 0.3 mm, spaced 2.16 mm from one another in the streamwise direction, for two flow velocities: 9.4 and 35.2 cm/sec. The points were located on the centerline of a rectangular channel with cross section 12.5 × 28.6 mm.

We see from the oscillogram that the signals at the two points are highly correlated, and the signal from the downstream region lags by some time relative to the signal from the first region.

Figure 2a shows curves of estimates of the cross-correlation functions for four flow regimes in the same channel. The flow velocities are, respectively, 9.4, 17.6, 18.8 and 35.2 cm/sec (Fig. 1 shows the signals for flows 1 and 2).

We see that increase of the flow velocity leads to reduction of the time shift corresponding to the peak of the cross-correlation function. The position of this peak can be used to determine the average flow velocity.

The average velocity values obtained by the correlation method agree to within ±2.7% with the values found by the frequency method [11] and those found by measuring the water discharge rate; the latter values were determined to within ±2%.

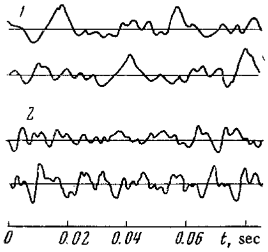

Fig. 1

Curve 3 in Fig. 2a corresponds to subcritical flow, and curve 4 is for supercritical flow. We see from these curves that the small change of the average velocity upon transition through the critical value alters markedly the frequency of the periodic component of the cross-correlation function.

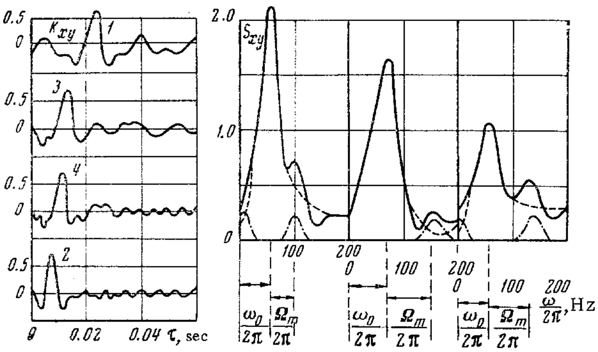

Fig. 2

The cross-correlation-function spectra $S_{xy}(\omega)$ for the first, third, and fourth flow regimes are also shown in Fig. 2b (solid curves). Also shown are the curves of the structure-function spectrum $S_\varepsilon(\omega)$ (dashed curves). As we noted above, the difference of the $S_{xy}(\omega)$ and $S_\varepsilon(\omega)$ spectra yields the convolution

$$I(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} S_v(\Omega) S_\varepsilon(\omega - \Omega) \, d\Omega \qquad (5.1)$$

The values of $I(\omega)$ found are shown by the dash-dot curves in the same figure.

As a result of its smallness, the modulation factor can be found from the relationship between the maximum amplitudes of the carrier frequency spectrum $S_\varepsilon(\omega)$ and the side component spectrum $I(\omega)$. The ratio of the frequency deviation to the carrier frequency yields the degree of turbulence, amounting to 7, 6, and 20% for the three given regimes.

Solving (5.1) for $S_v(\Omega)$, we find the velocity pulsation spectrum (Fig. 3). We see from Fig. 3 that for low-velocity flow (regime 1) the velocity pulsation spectrum is narrow and the oscillation energy is small. With increase of the average stream velocity the spectrum broadens, although the oscillation energy remains small in subcritical flow (regime 3). On transition from subcritical flow to supercritical flow, the oscillation energy increases sharply and the spectrum broadens (regime 4).

The velocity pulsation spectra measured by the correlation method were compared with the spectra obtained by an inductive sensor [12] and a hot-wire anemometer [13]. The sensitivity of the latter instruments was considerably lower than in the measurements by the correlation method; therefore comparison was possible only for flows with high velocities. The results obtained by all these methods agreed to within ±15%.

The inductive sensor consists of a pair of 0.5-mm-diameter electrodes, spaced 0.6 mm apart and located in the field of a permanent magnet (magnet is outside the flow). The sensitive element of the hot-wire anemometer was made in the form of a quartz filament 4 mm long and about 0.02 mm in diameter.

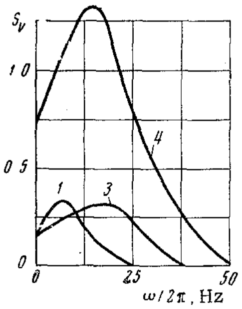

Fig. 3

Similar results were also obtained in a study of gaseous flows. Thus, in [14] a conventional schlieren system was used to obtain optical signals from two points in a subsonic cold air jet. Optical signature of the flow in the axial section was provided by a fine heated filament located at a considerable distance upstream in order to identify the signals from a region which was small in relation to the jet diameter. The nature of the recorded signals in this case is similar to that shown in Fig. 2; however, the signal spectrum and the cross-correlation-function spectrum are shifted into the higher frequency region (on the order of one kilohertz) as a result of the higher stream velocity Comparison of these results with the hot-wire anemometer measurements shows similar values.

Of interest is the application of this method to study of supersonic plasma streams, where it makes it possible to find both the velocity pulsations and the local value of the Mach number.

In fact, the distance traveled by the acoustic wave, which is the optical nonhomogeneity, relative to the recording device during the time t can be determined:

$$\xi(t) = \langle v \rangle t + \int_0^t v'(t)\,dt + \int_0^t c(t)\,dt \tag{5.2}$$

where c(t) is the velocity of the nonhomogeneity relative to the stream.

Assuming v'(t) and c(t) statistically independent, we can find the variance:

$$\langle \xi^2(t) \rangle = 2K_v(0)\int_0^t (t-\tau)\,R_v(\tau)\,d\tau + 2K_c(0)\int_0^t (t-\tau)\,R_c(\tau)\,d\tau \tag{5.3}$$

Consequently the maximum deviation of the coordinate from its average value at any time t will be defined by the quantity

$$\sigma_{\max} = \pm\sqrt{K_v(0) + K_c(0)}\,t \tag{5.4}$$

and the frequency deviation

$$\Delta\omega_\nu = \frac{2\pi}{\lambda_\nu}\sigma_{\max} = \pm\frac{\omega_\nu}{\langle v \rangle}\sqrt{K_v(0) + K_c(0)} \tag{5.5}$$

This expression can be reduced to the form

$$\frac{\Delta\omega_\nu}{\omega_\nu} = \pm\left[\frac{K_v(0)}{\langle v \rangle^2} + \frac{K_c(0)}{\langle v \rangle^2}\right]^{1/2} = \pm\left[\delta^2 + \frac{1}{M^2}\right]^{1/2} \tag{5.6}$$

where δ is the degree of turbulence and M is the Mach number.

Measurements made in supersonic argon plasma flow [6] have shown that the use of correlation analysis makes it possible to find both δ and M, since the velocity pulsations modulate identically all the frequencies of the signals being recorded, while modulation resulting from the relative motion of the acoustic waves affects only the high frequencies.

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